FUZZY HYBRID COMPUTING IN CONSTRUCTION ENGINEERING AND MANAGEMENT: THEORY AND APPLICATIONS

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FUZZY HYBRID COMPUTING IN CONSTRUCTION ENGINEERING AND MANAGEMENT: THEORY AND APPLICATIONS

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This book is dedicated to my son, Jack, whose life gives mine its greatest meaning.

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The editor would like to thank and acknowledge Sarah Miller for her assistance in compiling, proofreading and preparing the final manuscript. Her dedication and expertise proved invaluable.

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Foreword

This treatise is about a timely, important, and profoundly visible problem in construction engineering and management that can be solved with the aid of fuzzy sets and hybrid technologies.

With an increase in the complexity of systems and the associated problems with system analysis and synthesis, it is apparent that we are faced with the unavoidable issue of uncertainty. Information granularity – quite often formalised with the aid of fuzzy sets – supports various ways of representing and managing the uncertainty inherent in various branches of science and engineering. Construction engineering, with all its accompanying dimensions of complexities of management, is a visible and compelling example of where the benefits of the technology derived from fuzzy sets become tangible.

To take advantage of what information granules have to offer, a prudent formalisation of information granularity is required. There are many well-established directions that have been explored in the research, including probability, set theory (interval calculus), rough sets, fuzzy sets and random sets. Among these, fuzzy sets have established themselves as one of the most visible formalisms, and they have demonstrated several well-delineated advantages. The very nature of data, along with the structuralisation of experts' knowledge and fuzzy sets' abilities to cope with linguistically conveyed tidbits, are areas where fuzzy sets have shown their potential.

Fuzzy sets usually come hand in hand with other computational intelligence technologies, especially neurocomputing. Neural networks and fuzzy sets are highly complementary, and together they fully address the fundamentals of learning and knowledge representation. Their synergy is not only beneficial, but also essential, because in today's world, applications are a necessity for delivering advanced and practically viable problem-solving approaches.

The following well-known adages – attributed to Marr, the pioneer in image understanding, and originating from computer vision – are descriptive of the situations encountered in various domains of decision-making. The *principle of least commitment* emphasises the fact that there needs to be an adequate amount of experimental evidence before any decision, action or classification can be realised. It is therefore necessary to quantify this evidence or flag a lack of knowledge. The *principle of graceful degradation* is, in essence, a reformulation of the quest to endow solutions with a significant level of robustness. The relevance of these principles is apparent in all situations where one is faced with many poorly defined objectives, requirements and constraints. Fuzzy sets have emerged as an ideal vehicle for making these principles implementable. There are numerous uncontrollable and not fully observable factors involved in decision-making processes, including human factors, ways of making judgements, methods of efficiently capturing domain knowledge and the expertise of professionals. All of these are a viable target of focused studies. They need to be studied, formalised and handled algorithmically if one wishes to arrive at meaningful and efficient real-world solutions.

This book is a well-balanced body of knowledge that covers the fundamentals of fuzzy sets in Part 1 and embraces the essentials of fuzzy sets – which are of visible relevance to any novice to the area – such as fuzzy set notions, logic operations and hybrid techniques. Part 2 includes a discussion on fuzzy arithmetic and an investigation into fuzzy simulation completed in the fuzzy set environment, which are important topics that deserve a great deal of attention considering the different approaches present in the existing literature. Fuzzy decision-making, with its fundamental ideas of fuzzy objectives, fuzzy constraints and consensus building, has been an area of intensive and fruitful study, and these topics are also authoritatively covered in Part 2. Part 3 is a testimony to the diversity of applications where fuzzy sets and their hybrid developments play a pivotal role. The spectrum of applied studies is remarkably broad and ranges from investment appraisal to risk modelling to construction management.

The editor, Dr Aminah Robinson Fayek, should be congratulated on putting forward a timely, important, and badly needed volume that delivers a holistic and systematic view of the state-of-the-art in the discipline. There is no doubt that this field of research and application will grow in importance, and the concepts, methodologies and algorithms presented in this volume in the area of construction engineering and management will also be of interest to those working in other engineering and management disciplines.

Witold Pedrycz
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Preface

Introduction

The construction industry is a vital part of many national economies, contributing to a significant proportion of the gross domestic product. Construction industry productivity and performance are largely dependent on the effective planning, execution and control of construction projects, which occur in an environment of complexity and uncertainty. Many of the decisions and processes involved in construction projects are complex in nature due to numerous interacting factors and sometimes multiple conflicting objectives. Large projects with long durations, especially, involve many different disciplines and competing stakeholder interests. The interacting factors that must be accounted for when making project management decisions are complicated by the involvement of human activities and subjective reasoning. Given the often unique nature of each construction project, choices must be made in an environment that is characterised by high degrees of uncertainty, where quick decisions by experts must be taken that are based on complex systems and imprecise or unstructured variables. Uncertainty in construction has traditionally been treated as a random phenomenon that requires sufficient numerical project data for effective modelling. However, in construction, it is often the case that numerical project data do not meet the standards of quantity or quality required for effective modelling, or the data might not be completely reflective of new project contexts. Furthermore, in addition to random uncertainty, subjective uncertainty exists in construction, stemming from the use of approximate reasoning and linguistically expressed expert knowledge, the latter of which is often not formally documented.

To address the challenges related to subjective uncertainty in construction, researchers have applied fuzzy logic to construction process modelling and decision-making. Fuzzy logic is an effective technique for modelling approximate reasoning and computing with linguistic terms; it provides a means to draw definite conclusions from ambiguous information and in the absence of complete and precise data. However, fuzzy logic alone has a number of limitations, primarily in its inability to learn from data and its extensive reliance on expert knowledge for the development of often context-dependent models. These limitations can be overcome by integrating fuzzy logic with other techniques that have complementary strengths, thus leading to advanced and powerful fuzzy hybrid computing techniques.

Although fuzzy logic and fuzzy hybrid computing have a long history of application in a broad range of disciplines, their application in construction engineering and management is relatively new. A review of the literature shows an increase in the application of fuzzy hybrid computing in construction research over the past decades, and research topics based on fuzzy hybrid computing in the construction domain have become highly diversified. *Fuzzy Hybrid Computing in Construction Engineering and Management: Theory and Applications* reflects the increase in both the number and diversity of studies in this area.

Purpose and Structure of the Book

This book presents an overview of some of the many state-of-the-art fuzzy hybrid computing techniques developed in the construction domain, and it illustrates how researchers have used these techniques to solve a wide variety of construction engineering and management problems. Each chapter identifies key trends and future areas for research and development. Authors from around the world have contributed to this book, bringing unique perspectives on how to integrate fuzzy logic with other techniques and how to apply the resulting fuzzy hybrid techniques to solve practical construction industry problems.

This book is a guide for students, researchers and practitioners to the latest theory and developments in fuzzy hybrid computing in construction engineering and management. By providing an introduction to the basic theory related to fuzzy logic, a survey of the literature in fuzzy hybrid computing for construction engineering and management and explorations of both methodological and applied approaches, this book is a valuable resource for readers of all levels of knowledge and experience. Experienced researchers can use this book as a reference to the state-of-the-art in fuzzy hybrid computing techniques in construction, including an up-to-date literature review and references to the latest studies. By reading this book, both undergraduate and graduate students will be introduced to the field of fuzzy hybrid computing and exposed to examples of the latest advancements and practical applications in this field. Construction industry practitioners can use the book to develop a body of knowledge about the field, identify solutions to problems they face and consider these novel approaches for solving construction-related problems.

This book is organised in three parts. Part 1 provides an introduction to fuzzy logic in the context of construction engineering and management, including its basic concepts and suitability for construction modelling. Part 1 also includes a survey of the latest research in fuzzy hybrid computing and its applications in the context of construction engineering and management. Part 2 is comprised of several methodological chapters in the theory of fuzzy hybrid computing. These chapters discuss fuzzy arithmetic, fuzzy simulation, fuzzy consensus, fuzzy aggregation and fuzzy multi-criteria decision-making approaches. They also provide in-depth knowledge of the implementation of these approaches in construction. Part 3 presents several practical applications of fuzzy hybrid computing techniques in construction, illustrating how many of the techniques presented in earlier chapters are applied to solve real-world problems in a wide range of situations.

Chapter Summaries

Part 1: Introduction to Fuzzy Logic and Overview of Fuzzy Hybrid Techniques in Construction Engineering and Management

Introduction to Fuzzy Logic in Construction Engineering and Management
Fayek and Lourenzutti present an introduction to fuzzy logic in construction engineering and management. The role of fuzzy logic in handling certain types of uncertainties that are common in construction problems – such as subjectivity, ambiguity and vagueness – is highlighted. The role of fuzzy logic in construction problems is contrasted with that of probability theory, showing the complementary link between both theories. The authors present the key definitions, properties and methods of fuzzy logic, including the definition and representation of fuzzy sets and membership functions, basic operations on fuzzy sets, fuzzy relations and compositions, defuzzification methods, entropy for fuzzy sets, fuzzy numbers, methods for the specification of membership functions and fuzzy rule-based systems. Lastly, the authors discuss some challenges that fuzzy methods alone cannot handle, illustrating the need for hybridisation with other techniques.

Overview of Fuzzy Hybrid Techniques in Construction Engineering and Management
Gerami Seresht, Lourenzutti, Salah and Fayek present an overview of common types of fuzzy hybrid techniques applied to construction problems between 2004 and 2018. The techniques are grouped into four main categories: fuzzy hybrid optimisation, fuzzy hybrid machine learning, fuzzy multi-criteria decision-making and fuzzy simulation. For each category of fuzzy hybrid technique, the limitations of the standard techniques for solving construction-related problems are discussed, and the ways in which these limitations are overcome by using fuzzy hybrid techniques are described. Papers were selected for review that illustrate the capability of these types of fuzzy hybrid techniques to address construction challenges in a variety of applications. Finally, some directions for future research are presented.

Part 2: Theoretical Approaches of Fuzzy Hybrid Computing in Construction Engineering and Management

Fuzzy Arithmetic Operations: Theory and Applications in Construction Engineering and Management
Gerami Seresht and Fayek discuss fuzzy arithmetic operations and their application in solving mathematical equations that include fuzzy numbers. They present the two approaches for implementing fuzzy arithmetic operations, the α-cut approach and

the extension principle approach. They illustrate both approaches using triangular fuzzy numbers, and they present computational methods for implementing both approaches. They provide an example of the application of fuzzy arithmetic operations in a construction earthmoving simulation, and they outline future areas of research to extend the computational methods presented.

Fuzzy Simulation Techniques in Construction Engineering and Management
Raoufi, Gerami Seresht, Siraj and Fayek present three different approaches for fuzzy simulation: fuzzy discrete event simulation, fuzzy system dynamics and fuzzy agent-based modelling. They present an overview of simulation techniques used in construction and the advantages of integrating fuzzy logic with simulation techniques in order to deal with subjective uncertainties in simulation modelling. They illustrate how fuzzy logic can be integrated with discrete event simulation, system dynamics and agent-based modelling to enhance the capabilities of each method and make them more suitable for construction modelling. They discuss the process of choosing a suitable fuzzy simulation technique based on the characteristics of the construction system being modelled, the features of the simulation technique and the abstraction level of the model. They then present different applications of fuzzy simulation techniques in construction, and they outline areas for future applications and development.

Fuzzy Set Theory and Extensions for Multi-criteria Decision-making in Construction Management
Chen and Pan present 19 different methods for fuzzy multi-criteria decision-making (FMCDM) in construction, two of which they improve upon. They discuss multi-criteria decision-making (MCDM) methods in the construction context, fuzzy sets and extensions of fuzzy sets. They illustrate how MCDM methods can be enhanced with the integration of fuzzy logic in order to deal with complex problems that involve diverse decision makers' interests, conflicting objectives and uncertain information. In addition to presenting theoretical formulations for FMCDM methods, they summarise recent applications of these techniques in construction management, and they present future research needs in the development and application of FMCDM in construction management.

Fuzzy Consensus and Fuzzy Aggregation Processes for Multi-criteria Group Decision-making Problems in Construction Engineering and Management
Siraj, Fayek and Elbarkouky present different fuzzy consensus-reaching processes and fuzzy aggregation methods that are applicable to multi-criteria group decision-making (MCGDM) problems in construction. They present the basic theory and formulation of these methods and provide numerical examples to illustrate the steps involved in applying them to MCGDM problems. They discuss the application of fuzzy consensus reaching and fuzzy aggregation in the construction domain and provide examples of various applications. Finally, they present areas of future work that highlight emerging trends and future needs in the development of fuzzy consensus-reaching and fuzzy aggregation methods to solve MCGDM problems in construction.

Fuzzy AHP with Applications in Evaluating Construction Project Complexity
Nguyen, Le-Hoai, Tran, Dang and Nguyen present an application of the fuzzy analytic hierarchy process (AHP) for evaluating construction project complexity. This fuzzy AHP is capable of accounting for the qualitative nature of the factors involved in assessing project complexity. The authors describe the components of fuzzy extensions of the AHP, and they discuss the challenges of combining fuzzy logic with the traditional AHP. They present an entropy-based fuzzy extension of the AHP and its application in the evaluation of construction project complexity, which is illustrated with a case study. They discuss future research needs related to both the fuzzy AHP and the analysis of construction project complexity.

Part 3: Applications of Fuzzy Hybrid Computing in Construction Engineering and Management

The Fuzzy Analytic Hierarchy Process in the Investment Appraisal of Drilling Methods
Tokede, Ayinla and Wamuziri describe an application of the fuzzy analytic hierarchy process (AHP) in assessing investment appraisal risks for oil drilling projects. They compare the fuzzy AHP approach to a Monte Carlo simulation approach using a case study, and they conclude that both give comparable assessments of the level of risk for different drilling options; however, the fuzzy AHP provides the advantage of being able to take into account qualitative criteria in addition to quantitative criteria. They discuss the advantages of using the fuzzy AHP in an environment characterised by subjective uncertainty and linguistic assessments, and they provide ideas for future applications of the fuzzy AHP in risk analysis.

Modelling Risk Allocation Decisions in Public–Private Partnership Contracts Using the Fuzzy Set Approach
Ameyaw and Chan present a methodology for calculating the risk management capabilities of public–private partnerships in order to reach better risk allocation decisions. The proposed methodology is based on integrating risk allocation decision criteria, the Delphi method and the fuzzy synthetic evaluation (FSE) technique, allowing decision makers to use linguistic evaluations in the assessment of risk management capabilities. The authors illustrate their methodology using empirical data collected through a three-round Delphi survey. They demonstrate how their methodology relies on clearly stated risk allocation criteria, rather than on decision makers' popular opinions and risk preferences. The authors then present future research directions for advancing and automating the proposed approach.

Flexible Management of Essential Construction Tasks Using Fuzzy OLAP Cubes
Marín Ruiz, Martínez-Rojas, Molina Fernández, Soto-Hidalgo, Rubio-Romero and Vila Miranda propose a fuzzy multi-dimensional data model and on line analytical processing (OLAP) operations to manage construction data and support the decision-making process based on previous experience. Their framework enables

the integration of data in a common repository and provides flexible structures for representing data in the main tasks of construction project management. Imprecision in construction data is handled by incorporating fuzzy methods in the framework, making the documentation and interpretation of such data more intuitive to users of the framework. Use of the framework is illustrated with a number of practical construction applications. The authors conclude with a discussion of future challenges in the fuzzy database domain.

Using an Adaptive Neuro-fuzzy Inference System for Tender Price Index Forecasting: A Univariate Approach

Oshodi and Lam present an application of an adaptive neuro-fuzzy inference system (ANFIS) to the problem of forecasting tender prices. They compare the performance of the ANFIS to a similar model developed using the Box-Jenkins method and one developed using a support vector machine (SVM), using a univariate modelling approach for all three models. The performance of the ANFIS model is found to be superior to the other two modelling approaches when compared to actual data in predicting a tender price index. They conclude that fuzzy hybrid modelling approaches, such as the ANFIS, show promise in accurately modelling nonlinear problems in construction engineering and management, and they give examples of construction-related problems that may benefit from the application of such approaches.

Modelling Construction Management Problems with Fuzzy Cognitive Maps

Case, Blackburn and Stylios use fuzzy cognitive maps (FCMs) to model construction management problems. They illustrate the development and use of FCMs in modelling the complex relationships of the numerous factors that impact the feasibility and performance of construction projects. Their approach incorporates fuzzy logic with cognitive maps to allow domain experts to define the cause and effect relationships between factors using linguistic terms. They describe how to develop FCMs for construction management problems and how they can be used to test various scenarios and make decisions in the context of cost, schedule and risk management. Finally, they propose extensions to their FCM approach for construction management.

Crane Guidance Gesture Recognition Using Fuzzy Logic and Kalman Filtering

Wang and Gordon propose a new approach to tracking and recognising human arm gestures for crane guidance on construction sites. The authors use data collected in real time from both a Kinect visual sensor and a Myo armband sensor to estimate Euler angles, angular velocity, linear acceleration and electromyography. Kalman filtering is applied for motion trajectory tracking, and a fuzzy inference system is used to interpret the crane operator's arm gestures. The methodology is illustrated in an experiment involving Kinect, the Myo armband and MATLAB/ Simulink software using five different signals for crane guidance, illustrating the effectiveness and robustness of the method in crane guidance applications. They propose future research to evaluate the robustness of their approach with an increase in the number of crane signals as part of automated crane control systems.

Future Directions

This book presents the latest advancements in both the theory and applications of fuzzy hybrid computing in construction engineering and management. It identifies emerging areas of inquiry and opportunities for future research and development. With the knowledge contained in this book, innovative solutions for problems facing the construction industry can be developed, helping this vital and important sector of the world economy thrive and become more profitable and competitive.

Some of the emerging areas of inquiry discussed in this book include:

(1) Improving methods of eliciting and aggregating expert knowledge, combining such knowledge with data-driven techniques, and integrating data in different formats for use in fuzzy hybrid systems. Capturing human expertise while simultaneously capitalising on the richness of data in different formats is essential for the development of fuzzy hybrid systems that are appropriate for the construction domain.
(2) Developing more robust and automated methods of specifying membership functions and determining the most appropriate fuzzy operations for fuzzy hybrid systems. Also discussed is the development of optimisation techniques for fuzzy hybrid systems that can help with selecting the best system configurations. Such research will reduce the amount of effort required to develop new systems for different applications.
(3) Developing methods of adapting and transferring fuzzy hybrid systems to contexts for which they were not developed in order to address the context-dependent nature of their application. These methods will reduce the effort required to develop a unique system for each new construction context.
(4) Identifying further opportunities to hybridise fuzzy logic with other techniques in order to create even more advanced fuzzy hybrid computing methods for dealing with different aspects of construction problems.
(5) Identifying new areas of application in construction engineering and management that would benefit from fuzzy hybrid modelling in order to provide practitioners with solutions to problems they face in the planning, execution and control of construction projects.

Furthermore, automating advanced fuzzy hybrid techniques in software platforms will make them more accessible to construction practitioners, who will not be required to have knowledge of the techniques on which the software is based. Such developments will facilitate more widespread acceptance and use of fuzzy hybrid techniques in construction practice.

I hope you find this book as interesting and thought-provoking as I have. It has been a great pleasure working with the many talented authors who have contributed their research and perspectives on fuzzy hybrid computing in construction engineering and management. We hope this book will be updated as the fuzzy logic and fuzzy hybrid computing community in construction continues to advance these techniques. With such advancements, we will find new ways of hybridising fuzzy logic with other techniques to develop innovative solutions to practical problems faced by construction industry practitioners, helping this important sector of the world economy become more technologically sophisticated, competitive and profitable.

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PART 1
INTRODUCTION TO FUZZY LOGIC AND OVERVIEW OF FUZZY HYBRID TECHNIQUES IN CONSTRUCTION ENGINEERING AND MANAGEMENT

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Introduction to Fuzzy Logic in Construction Engineering and Management

Aminah Robinson Fayek and Rodolfo Lourenzutti

Abstract

Construction is a highly dynamic environment with numerous interacting factors that affect construction processes and decisions. Uncertainty is inherent in most aspects of construction engineering and management, and traditionally, it has been treated as a random phenomenon. However, there are many types of uncertainty that are not naturally modelled by probability theory, such as subjectivity, ambiguity and vagueness. Fuzzy logic provides an approach for handling such uncertainties. However, fuzzy logic alone has some limitations, including its inability to learn from data and its extensive reliance on expert knowledge. To address these limitations, fuzzy logic has been combined with other techniques to create fuzzy hybrid techniques, which have helped solve complex problems in construction. In this chapter, a background on fuzzy logic in the context of construction engineering and management applications is presented. The chapter provides an introduction to uncertainty in construction and illustrates how fuzzy logic can improve construction modelling and decision-making. The role of fuzzy logic in representing uncertainty is contrasted with that of probability theory. Introductory material is presented on key definitions, properties and methods of fuzzy logic, including the definition and representation of fuzzy sets and membership functions, basic operations on fuzzy sets, fuzzy relations and compositions, defuzzification methods, entropy for fuzzy sets, fuzzy numbers, methods for the specification of membership functions and fuzzy rule-based systems. Finally, a discussion on the need for fuzzy hybrid modelling in construction applications is presented, and future research directions are proposed.

Keywords: Fuzzy sets; fuzzy logic; uncertainties; construction; fuzzy numbers; fuzzy hybrid techniques

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List of Notations

$\mathbb{R}^+$	set of positive real numbers
A_α	α-cut of the fuzzy set A
A_{α^+}	strong α-cut of the fuzzy set A
α	membership grade
A^c	the complement of a set A
$L_\alpha^{(c)}$	lower bound of the α-cut of the cth expert
$LH_\lambda(\cdot)$	linguistic hedge modifier
$S_b(.,.)$	bounded sum
$S_d(.,.)$	drastic union
$S_m(.,.)$	standard union
$S_s(.,.)$	algebraic sum
$T_L(.,.)$	bounded difference
$T_m(.,.)$	standard intersection
$T_p(.,.)$	algebraic product
$T_d(.,.)$	drastic product
$U_\alpha^{(c)}$	upper bound of the α-cut of the cth expert
w_c	weight of the cth expert
$\hat{y}$	output of fuzzy rule-based system
$h(\cdot)$	the height of a fuzzy set
$H(\cdot)$	entropy of a fuzzy set
M	space of membership functions
$R \circ Q$	composition of binary relations R and Q
$S(.,.)$	s-norm operator
$T(.,.)$	t-norm operator
X	universe of discourse
x	an element of X
$\mu(\cdot)$	membership function
ζ	inconsistency index
ν	centres of the clusters
E_f	expectation with respect to probability distribution f
J_m	objective function of fuzzy c-means
P	pairwise comparison matrix of the AHP

$p_{(ij)}$	an element of matrix P
$\lambda_{(max)}$	highest eigenvalue of P

Fuzzy Logic for Handling Uncertainty in Construction Engineering and Management

The construction industry's value to society is significant. The places where people live and work and the transportation infrastructure that allows them to travel between destinations are all realised by the construction sector. Because of its inescapable presence, it is not surprising that the construction sector is also a major contributor to economic growth. It is critical to any national economy, contributing significantly to gross domestic product and impacting all areas of society. The construction industry often faces challenges to its future growth, particularly in times of economic uncertainty. Lower-than-expected productivity, high project costs and a need to efficiently use scarce resources often threaten investment in construction. These obstacles stem from the high risk and uncertainty that are an inevitable challenge for the industry. Construction management involves the development and application of techniques that will improve our ability to plan, structure, forecast, control and evaluate projects in order to deliver results that meet or exceed performance objectives, such as time, cost, productivity, quality and safety. Construction projects frequently represent large investments from numerous parties, but modelling and predicting the ways in which they will evolve is a difficult task. Decisions related to project planning, execution and investment, for example, can be quite complex. This complexity arises from the multiple interacting factors that affect each project and the fact that each project is unique, limiting the direct transfer of knowledge and data from previous projects that could be used to predict the ways in which a future project will unfold. Hence, there is a heavy reliance on experts to make decisions in the construction industry, which causes a demand for modelling techniques that are able to capture and process subjectivity.

Several models and systems have been developed that can deal with the high level of uncertainty and subjective reasoning involved in construction projects. However, in construction, uncertainty has traditionally been treated as a random phenomenon, frequently requiring sufficient historical numerical project data for effective modelling; these data are often not available nor are they reflective of the current situation. Additionally, there are types of uncertainties, such as ambiguity, subjectivity and vagueness, that are not naturally modelled as random phenomena. These uncertainties are present in most decision-making processes, and still have not been effectively captured and harnessed in construction decision-making systems. In this context, the use of fuzzy sets and fuzzy logic becomes crucially useful.

Introduced by Zadeh (1965), fuzzy set theory transformed the way that uncertainties are modelled. The new theory of fuzzy sets caused some resistance, especially among statisticians (Kandel, Martins, & Pacheco, 1995; Laviolette, Seaman, Barret, & Woodall, 1995a; Laviolette, Seaman, Barrett, & Woodall, 1995b; Lindley, 1987). For example, Tribus (1979) questioned whether the theory of fuzzy sets brought advantages compared to Bayesian methods for solving real problems. In the same reference, Kandel replied, stating that it is not about the theory of fuzzy sets being always useful, but rather about fuzzy sets being an available and viable tool for researchers (Tribus, 1979). Zadeh (1980) joined the discussion by claiming that in most cases of practical interest, both probabilistic and fuzzy theories must be combined to achieve realistic solutions. Later, Laviolette and Seaman (1994) rejected the reasoning that probability theory is not able to model the uncertainties captured by fuzzy sets, and they stated that 'probability theory provides a completely and uniquely optimal means for solving problems and managing uncertainty' (Laviolette & Seaman, 1994, p. 14). However, the theory of fuzzy sets has been intensively investigated over the years, leading to the development of a variety of different fuzzy techniques in diverse fields of study, such as learning (Jiang, Deng, Chung, & Wang, 2017; Singh, Pal, Verma, & Vyas, 2017; Zuo, Zhang, Pedrycz, Behbood, & Lu, 2017), quality control (Kaya, Erdogan, & Yıldız, 2017; Kaya & Kahraman, 2011; Şentürk, Erginel, Kaya, & Kahraman, 2014) and decision-making (Lourenzutti & Krohling, 2016; Roszkowska & Kacprzak, 2016; Tyagi, Agrawal, Yang, & Ying, 2017). In fact, the theory of fuzzy sets has also heavily contributed to research in the construction industry (Cheng & Hoang, 2015; Elbarkouky, Fayek, Siraj, & Sadeghi, 2016; Mirahadi & Zayed, 2016). This widespread application of fuzzy sets shows the unquestionable usefulness of fuzzy set theory. Additionally, in many works (e.g. Li, Wang, & Geng, 2017; Lourenzutti, Krohling, & Reformat, 2017; Tang, Chen, Hu, & Yu, 2012), both fuzzy sets and probability are used, endorsing the point of view of Zadeh (1980, 1995) that fuzzy sets and probability are complementary.

As fuzzy sets extended the notion of classical sets, classical logic (or Boolean logic) was also extended to handle fuzzy sets. This new approach, named fuzzy logic, is a precise and powerful technique that is able to handle natural language and approximate reasoning, mathematically translating linguistic variables into numeric form and allowing us to draw definite conclusions from ambiguous information and in the absence of complete and precise data (Klir & Yuan, 1995; Pedrycz & Gomide, 2007; Ross, 2010; Zadeh, 1965). It provides us with a technique that handles certain types of uncertainties more naturally than probabilistic models, enabling us to build vastly improved models of human reasoning and expert knowledge that are ideal for use in construction management applications (Chan, Chan, & Yeung, 2009; Fayek & Oduba, 2005; Plebankiewicz, 2009).

Fuzzy logic is an ideal technique for dealing with certain characteristics of construction-related problems, such as inexact input and output; the use of heuristic reasoning based on experience and judgement rather than algorithms; the need to make quick decisions that are often based on qualitative information (e.g. good ground conditions, bad weather); when more than one answer or solution is possible, which occurs in most cases where there is no optimal or exact solution; solutions that combine a large body of expert knowledge with subjective and sometimes contradictory opinions; the uniqueness of each project or process, which requires a new set of input variables and decision support tools that can capture variability in the absence of adequate historical data; the presence of dynamic conditions that are difficult to replicate; and a lack of continuity and proper transfer of knowledge and skills between construction personnel, necessitating the use of systems that capture and document expert knowledge. Fuzzy logic also has the capacity to take into account the underlying characteristics of a project that cannot be measured in certain terms and that are frequently ignored, such as worker skill and motivation, the quality of project teams and the quality and comprehensiveness of project practices. Models based on fuzzy logic can make the construction decision-making process more transparent, and they allow experts to express themselves in linguistic terms rather than strictly in numerical terms, which better suits their thought processes. However, models based on fuzzy logic alone have the following limitations: an inability to learn from data, extensive reliance on expert knowledge, a context-dependent nature and a lack of capacity for generalisation. Integrating fuzzy logic with other techniques can produce new approaches with the functionality necessary to overcome the limitations of each individual technique. The need for fuzzy hybrid modelling in construction is discussed in the Section Fuzzy Hybrid Modelling in Construction. Researchers have produced a number of practical, industry-relevant applications of fuzzy logic and fuzzy hybrid modelling, which are discussed in the chapter, 'Overview of Fuzzy Hybrid Techniques in Construction Engineering and Management'.

Fuzzy Sets and Membership Functions

In several situations, concepts that are not precisely defined must be addressed. This lack of precision makes the processing of such concepts much more complex, since they do not have a clear boundary between true and false. For instance, if a $200,000 house is considered *expensive*, a discount of $10 in the price of the house will probably not change its classification, and the house will probably still be considered *expensive*, but maybe to a lesser degree. The classification of a person as *tall* will probably not change if his or her size varies by 1 cm. Dealing with these uncertain concepts, or fuzzy concepts, using classical set theory may lead to unrealistic situations where, for example, a house that costs $199,999.99 is classified as having a *low* price while a house that costs $200,000 is classified as having a *medium* price. This issue occurs because, in classical set theory, an element either fully belongs or does not belong to a set. Such a requirement imposes a sharp boundary on uncertain concepts. On the other hand, fuzzy set theory, which can be seen as a generalisation of classical set theory, provides a way to overcome such challenges by allowing an element to partially belong to a set. This capability is accomplished by using a membership function.

Membership functions assign to each element a membership degree, usually denoted by μ, between 0 (no membership) and 1 (full membership). By using membership functions, it is possible to capture the gradual transition between concepts. For instance, the price of a house is measured numerically in dollars; however, the concepts of *low* and *medium* prices are not precisely defined, presenting a gradual transition from one concept to the other. In other words, a price, say of $150,000, can be considered a *low* price and a *medium* price at the same time but to

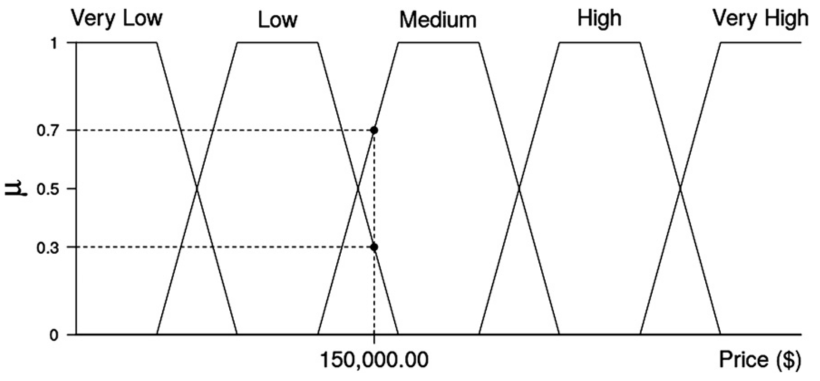

Figure 1: Membership Functions Representing Different Linguistic Concepts Related to Housing Prices in Dollars.

different extents. Figure 1 illustrates the gradual transition between concepts using a trapezoidal shape for membership functions, where $150,000 belongs to the concept of *low* with a membership degree of 0.3, and it also belongs to the concept of *medium* with a membership degree of 0.7. Fuzzy concepts are context dependent. For example, a value of x dollars for a house in a poorly developed area can be considered a *very high* price, while in a well-developed area it would represent a *very low* price.

When examining a specific problem, there are certain elements that are relevant and that must be taken into consideration. The set of all those elements is usually called the universe of discourse. The universe of discourse, contrary to what one may expect, is about narrowing down our attention to the relevant elements only. For example, when estimating the cost of a construction project (in dollars), one may focus only on positive numbers, therefore restricting the universe of discourse to $\mathbb{R}^+$. Or even further, one could define the universe of discourse as an interval $[a, b] \subset \mathbb{R}^+$, where a and b represent the lower and upper bounds (in dollars) for the cost of the project, respectively. In cases where the variable under consideration is qualitative, one might use auxiliary quantitative attributes to describe it. For instance, when evaluating the quality of a crew supervisor, one might take into account the years of experience of the supervisor (Knight & Fayek, 2002). By doing so, the universe of discourse that was originally qualitative is now quantitative. The universe of discourse is important because fuzzy sets are defined over a specific universe of discourse, that is, fuzzy sets are subsets, or better yet, fuzzy subsets, of a specific universe of discourse. Therefore, one must define the behaviour of a fuzzy set only over the elements of the universe of discourse, being aware that any value outside the universe of discourse is out of scope.

Now, let X be a universe of discourse. In classical set theory, every element $x \in X$ either belongs or does not belong to a set A. Consequently, the membership function of A, also called the characteristic function, denoted by μ_A, is either 0 if the element does not belong to A or 1 if it does, that is μ_A: $X \rightarrow \{0, 1\}$. On the other hand, the membership function of a fuzzy set *B*, denoted by μ_B, is defined as μ_B: $X \to [0, 1]$. Any element $x \in X$ can have a partial membership degree between 0 (does not belong to the set) and 1 (fully belongs to the set). Thus, it is possible that an element *x* belongs simultaneously to a set *A* and its complement, A^c (which means *not A*). For example, it is possible for a house to be classified as having a *low* price and at the same time be classified as *not* having a *low* price.

In the construction context, membership functions play an important role in representing most variables used in decision-making processes. For example, the productivity of a crew can be affected by a number of variables, such as crew size, crew skill, weather conditions and quality of supervision. Some of these variables, including crew size, are quantitative (i.e. with a numerical universe of discourse), and others, including quality of supervision, are qualitative (i.e. with a universe of discourse containing non-numerical elements).

Representing Membership Functions

The definition of fuzzy sets imposes only one restriction on membership functions, which is that they must lie on the interval [0, 1]. Thus, we can have unlimited variations of membership functions with many different shapes. It is important to have a practical way to represent such functions. In the case of a discrete universe of discourse *X*, the membership of a fuzzy set *A* is usually expressed as follows:

$$A = \sum_{i=1}^{n} \mu_i/x_i = \mu_1/x_1 + \mu_2/x_2 + \ldots + \mu_n/x_n = \sum \mu_A(x)/x, \tag{1}$$

where $\mu_i = \mu(x_i)$. For example, suppose that the variable *crew size* is being investigated and the universe of discourse is $X = \{1, 2, 3, 4, 5, 6\}$, meaning that *crew size* is a discrete variable. A discrete fuzzy concept, say *small*, can be represented as $small = \{1/1 + 1/2 + 1/3 + 0.7/4 + 0.4/5 + 0/6\}$. On the other hand, a continuous fuzzy set *A* can be expressed as

$$A = \int_i^n \frac{\mu_i}{x_i} = \frac{\int \mu_A(x)}{x}. \tag{2}$$

Although there are a wide range of membership functions, some specific shapes are frequently used, especially for continuous cases. One of the most used shapes of membership functions is the trapezoidal membership function (Dubois & Prade, 1978, 1980), which can be represented by five parameters (a, b, c, d, e) and is given by

$$\mu(x) = \begin{cases} \dfrac{(x-a)e}{b-a}, & \text{when} \quad a \le x < b \\ e, & \text{when} \quad b \le x < c \\ \dfrac{(d-x)e}{d-c}, & \text{when} \quad c \le x \le d \\ 0, & \text{otherwise} \end{cases}, \tag{3}$$

where $a \leq b \leq c \leq d$ and $0 < e \leq 1$ stands for the maximum membership. The parameter e is frequently assumed to be one. A trapezoidal membership function is illustrated in Figure 2.

A special case of trapezoidal membership functions occurs when $b = c$, resulting in triangular membership functions. Also, if $b - a = d - c = \delta$, the trapezoidal membership is deemed symmetric and can be represented by only four parameters (b, c, e, δ), otherwise it is asymmetric. Another important shape for continuous fuzzy sets is the bell-shaped (Gaussian) membership function specified by three parameters (a, b, e) and defined as

$$\mu(x) = e \times \exp\left\{ -\frac{(x-a)^2}{b} \right\}, \tag{4}$$

where $a \in \mathbb{R}$, $b > 0$ and $e \in (0,1]$. Figure 3 illustrates the bell-shaped membership function.

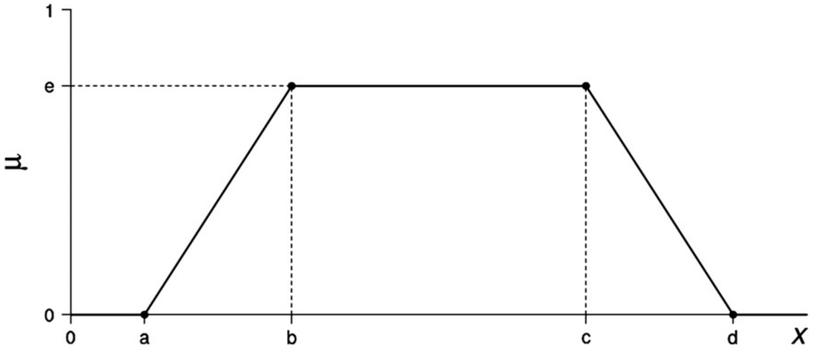

Figure 2: Trapezoidal Membership Function.

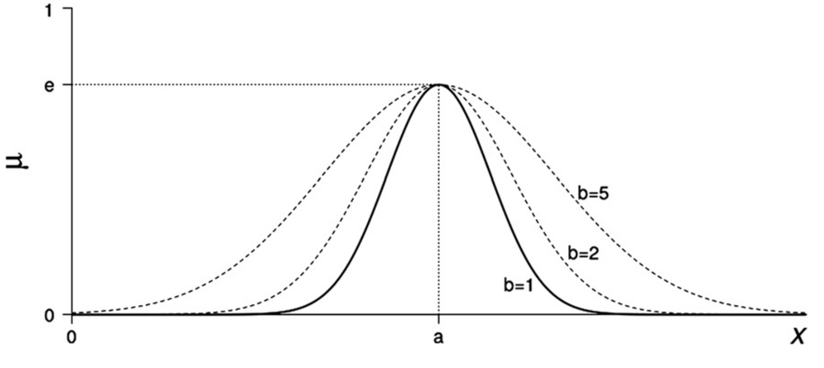

Figure 3: Bell-shaped (Gaussian) Membership Function.

Characteristics of Membership Functions

For the definitions presented in this section, consider a fuzzy set A with membership function μ_A, defined in a universe of discourse X, that is μ_A: $X \rightarrow [0, 1]$. A fuzzy set has three basic characteristics: support, height and core. The support of a fuzzy set is the crisp set of all elements of X that have a non-zero membership grade, that is

$$\mathrm{SUPPORT}(A) = \{x \in X : \mu_A(x) > 0\}. \tag{5}$$

The height, denoted by h, is the largest membership grade obtained by any element in X, in other words,

$$h(A) = \sup_{x \in X} \mu_A(x), \tag{6}$$

where sup denotes the supremum. If $h(A) = 1$, that is, if there exists at least one element $x \in X$ such that $\mu_A(x) = 1$, then the fuzzy set A is said to be normal; otherwise it is subnormal. In subnormal cases, it is very common to normalise the membership function to guarantee that the height is equal to 1. Lastly, the core of a fuzzy set is the crisp set of all elements that have a membership degree of 1, or mathematically,

$$\mathrm{CORE}(A) = \{x \in X : \mu_A(x) = 1\}. \tag{7}$$

Figure 4 illustrates these concepts in the case of a trapezoidal membership function. For trapezoidal membership functions, the support is the interval (a, d), the height is defined by the parameter e, and the core is the interval $[b, c]$.

Another important characteristic of fuzzy sets is the α-cut representation, which provides a connection between classical set theory and fuzzy set theory. An α-cut, defined at a membership grade α of a fuzzy set A, denoted by A_α, is a crisp set that

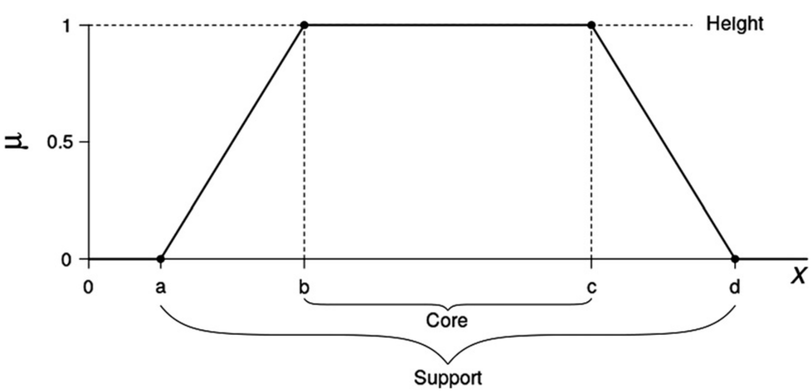

Figure 4: The Core, Support and Height of a Trapezoidal Membership Function.

contains all elements of the universe of discourse whose membership degrees are greater than or equal to the specified value of α. In other words,

$$A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}, \alpha \in (0, 1]. \tag{8}$$

Similarly, we have the strong α-cut, denoted by A_{α^+}, whose elements have membership degrees strictly greater than α, that is,

$$A_{\alpha^+} = \{x \in X : \mu_A(x) > \alpha\}, \alpha \in [0, 1). \tag{9}$$

Note that any fuzzy set can be completely represented by its α-cuts, where for any element $x \in X$, we have

$$\mu_A(x) = \sup_\alpha \alpha \mu_{A_\alpha}(x). \tag{10}$$

Since A_α is a crisp set, $\mu_{A_\alpha}(x)$ is either 0 or 1. Consequently, the membership of an element $x \in X$ is the highest membership grade for which the element still belongs to the α-cut set. For example, consider the concept *medium* for the amount of time in months to complete a construction project as a trapezoidal fuzzy number *medium* = $(0, 3, 7, 10, 1)$. The values with the highest membership degree of A are given by the crisp set $A_{1.0} = [3, 7]$, which is the core of A, while the values with a membership degree of at least 0.5 are in $A_{0.5} = [1.5, 8.5]$. Both α-cuts are illustrated in Figure 5. However, while $x = 1.5$ and $x = 8.5$ belong to $A_{0.5}$, for any small ϵ such that $0 < \epsilon \leq 0.5$, these two elements are not part of $A_{0.5+\epsilon}$. Consequently, $\alpha = 0.5$ is the highest membership grade for which those elements belong to A_α. Then, from Eq. (10), it is possible to conclude that $\mu_A(1.5) = \mu_A(8.5) = 0.5$.

Next, a fuzzy set A is convex if for any two elements $x_1, x_2 \in X$ and $0 \leq u \leq 1$, we have that $\mu_A(ux_1 + (1-u)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$ (Zadeh, 1965). In order words, for a fuzzy set to be convex, it is necessary that the membership degree between any two points is greater than the smaller membership degree of the two points. Figure 6 illustrates a non-convex fuzzy set. Note that if A is a convex fuzzy set, then

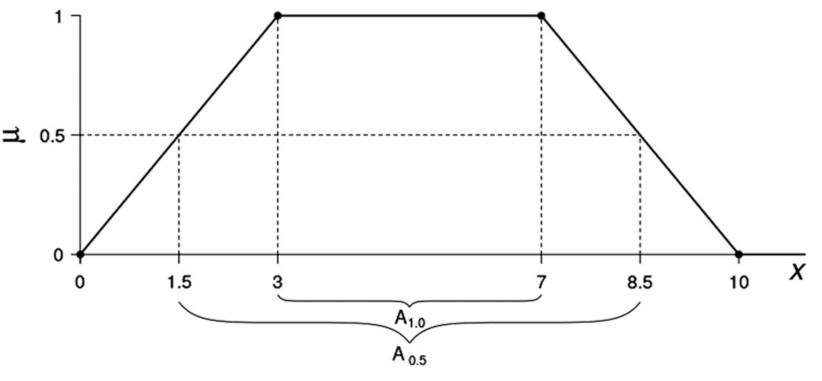

Figure 5: α-cuts of a Trapezoidal Membership Function.

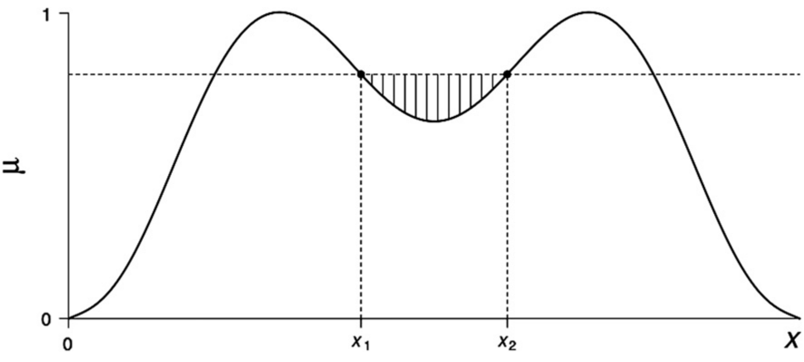

Figure 6: An Example of a Non-convex Fuzzy Set.

for any $\alpha \in (0, 1]$, $0 \le u \le 1$ and any $x_1, x_2 \in A_\alpha$, we have that $\mu_A(ux_1 + (1-u)x_2) \ge \min\{\mu_A(x_1), \mu_A(x_2)\} \ge \alpha$ and consequently, $ux_1 + (1-u)x_2 \in A_\alpha$. That is to say, if A is a convex fuzzy set, all its α-cut sets are convex sets. In cases where a classical property of a crisp set is generalised to fuzzy sets by requiring that all α-cut sets of a fuzzy set satisfy such a property, we say that the property is *cutworthy*.

Fuzzy Variables and Fuzzy Partitions

Fuzzy variables were first introduced by Zadeh (1975). A fuzzy variable is a variable for which the possible values, or levels, are fuzzy sets. For example, when evaluating the cost of a construction project, we can describe it as *small*, *medium* or *large*, which are linguistic terms defined over the universe of discourse. It is true that the underlying measure of cost is numerical, but the idea is to evaluate the compatibility of this underlying quantity with fuzzy concepts, which are the levels of the fuzzy variable cost, in this case, *small*, *medium* or *large*. While one may wonder about the necessity to transform precise data, such as the cost of \$20, 253, 086.32, into a much less precise concept, such as *small*, it is important to note that when modelling complex systems, we are rarely able to obtain such precise data, requiring the use of less precise concepts to cope with the complexity of the problem. This incompatibility between precision and complexity is known as the principle of incompatibility (Zadeh, 1973, 1975).

The concepts defined by the levels of a fuzzy variable are usually very context dependent. For example, if we are dealing with a commercial construction project, one that costs \$5 million may be considered a *large* project, but if we are dealing with an industrial construction project, this could be considered a *small* project, since industrial projects frequently cost billions of dollars. Even in commercial construction projects, if we focus our attention on a specific situation, such as the construction of shopping malls, a project that costs \$5 million may be considered a *small* one.

Now, let $A = \{A_1, \ldots, A_m\}$ be a fuzzy variable with m levels on X and let μ_i be the membership function of A_i. We say that A forms a fuzzy partition of X if $\sum_{i=1}^{m} \mu_i(x) = 1, \forall x \in X$, and for every $x \in X$, $\mu_i(x) > 0$ for at least one $i \in \{0, \ldots, m\}$. In other words, the sum of the membership degrees across all the partitions of any element in X is equal to 1, and all the partitions contain at least one element $x \in X$, even if with a low degree of membership. Figure 1 shows an example of a fuzzy partition. Note that if μ_i: $X \rightarrow \{0, 1\}$, $i = 1, \ldots, m$, then the fuzzy partition becomes a crisp partition.

Basic Set Operations on Fuzzy Sets

As in the case of crisp sets, there are a number of operations that can be performed on fuzzy sets, including the commonly used complement (representing the linguistic term *not*), the intersection (representing the linguistic term *and*), and the union (representing the linguistic term *or*). Originally, for two fuzzy sets A and B, Zadeh (1965) defined the three operations as follows:

(1) The complement A^c of A is given by $\mu_{A^c}(x) = 1 - \mu_A(x)$;
(2) The intersection of two fuzzy sets A and B is defined as $\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$; and
(3) The union of two fuzzy sets A and B is given by $\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$.

These are frequently called the standard operations.

Later, several different operators were proposed. In general, it was suggested that intersection operations should be performed using triangular-norm operators, denoted by t-norm, and union operations should be performed using triangular-conorm (t-conorm), also called s-norm, operators (Gupta & Qi, 1991, and references therein). A t-norm operator T :$[0, 1]^2 \rightarrow [0, 1]$ must satisfy the following properties:

(1) $T(x, y) = T(y, x)$;
(2) $T(x, 1) = x$;
(3) $T(x, T(y, z)) = T(T(x, y), z)$; and
(4) $T(x, y) \leq T(x, z)$, if $y \leq z$.

Common t-norms are:

(1) standard intersection: $T_m(x, y) = \min\{x, y\}$;
(2) algebraic product: $T_p(x, y) = xy$;
(3) bounded difference (or Lukasiewicz t-norm): $T_L(x, y) = \max\{0, x + y - 1\}$; and
(4) drastic product: $T_d(x, y) = \begin{cases} \min\{x, y\}, \text{if } \max\{x, y\} = 1 \\ 0, \text{otherwise} \end{cases}$.

In general, any t-norm T is bounded by the drastic product and the standard intersection; in fact, it is possible to show that $T_d(x, y) \leq T_L(x, y) \leq T_p(x, y) \leq T_m(x, y), \forall x, y$ (Hanss, 2005).

On the other hand, an s-norm operator S must satisfy the following properties:

(1) $S(x, y) = S(y, x)$;
(2) $S(x, 0) = x$;
(3) $S(x, y) \leq S(x, z)$, if $y \leq z$; and
(4) $S(x, S(y, z)) = S(S(x, y), z)$.

Common s-norms are:

(1) standard union: $S_m(x, y) = \max\{x, y\}$;
(2) algebraic sum: $S_s(x, y) = x + y - xy$;
(3) bounded sum: $S_b(x, y) = \min\{1, x + y\}$; and
(4) drastic union: $S_d(x, y) = \begin{cases} \max\{x, y\}, \text{if } \min\{x, y\} = 0 \\ 1, \text{otherwise} \end{cases}$.

Similarly, it is possible to show that $S_m(x, y) \leq S_s(x, y) \leq S_b(x, y) \leq S_d(x, y), \forall x, y$ (Hanss, 2005).

Another important class of operators is called *linguistic hedges*. Linguistic hedges modify the concept represented by a fuzzy set by changing the membership degree of its elements, resulting in a different concept that is still related to the original. Linguistic hedges are words that modify the original meaning of a concept, such as *very, extremely, fairly, somewhat* or *slightly*. For example, say that the amount of time in months to finish a project is *low*, which is represented by a fuzzy set. Suppose that $x = 2$ months has a membership degree of $\mu_{low}(2) = 0.8$. Since $x = 2$ satisfies the concept *low* only by 0.8, it is expected that $x = 2$ satisfies the concept of *very low* to a lesser degree, say $0.8^2 = 0.64$. On the other hand, if one is considering the concept *slightly low*, it is expected that $x = 2$ presents a higher level of membership, say $\sqrt{0.8} = 0.8944$. Note that some linguistic hedges, such as *very* and *extremely*, restrict the target concept by making it more difficult to be satisfied (i.e. by potentially decreasing the membership degree of its elements). These linguistic hedges are called strong modifiers. Linguistic modifiers such as *fairly, slightly* or *somewhat* broaden the target concept, making it easier to satisfy (i.e. by potentially increasing the membership degree of its elements). These linguistic hedges are called weak modifiers. For example, one might assume that a linguistic hedge yields the following change on the membership $LH_\lambda(\mu(x)) = \mu(x)^\lambda$; in this case, if $\lambda < 1$, LH_λ is a weak modifier (potentially increasing the membership degree) and if $\lambda > 1, LH_\lambda$ is a strong modifier (potentially decreasing the membership degree).

Fuzzy Relations and Fuzzy Composition

Let $X_1, \ldots, X_n$ be a collection of universes of discourse. A fuzzy relation R is a fuzzy set defined in $X_1 \times X_2 \times \ldots \times X_n$ with the membership function $\mu_R : X_1 \times \ldots \times X_n \rightarrow [0, 1]$ (Zadeh, 1965). Now, suppose that $R \subset X_1 \times X_2$ and $Q \subset X_2 \times X_3$ are two binary relations. Zadeh (1965) defined the composition of R and Q, denoted by $R \circ Q \subset X_1 \times X_3$, as

$$\mu_{R\circ Q}(x,z) = \sup_{y\in X_2} \min\{\mu_R(x,y), \mu_Q(y,z)\},\ x\in X_1 \text{ and } z\in X_3, \tag{11}$$

which is frequently called the *max-min* composition. Many different composition operators have been proposed (Lo, 1999; Pedrycz & Gomide, 2007). Note that in a fuzzy composition, one is relating two sets, X_1 and X_3, by a common relation they have with a third set, X_2. For example, say that the size of a project (X_1) is associated with the amount of time required to finish it (X_2); at the same time, the amount of time required to finish it (X_2) is related to crew costs (X_3). Then, we can relate the size of a project (X_1) to the crew costs (X_3) through their shared relation to the amount of time required to finish the project (X_2). In construction, fuzzy composition operations have been used to predict design cost overruns through a binary fuzzy relation between project characteristics and risk events and between risk events and design cost increases or decreases (Knight & Fayek, 2002).

Defuzzification

While fuzzy sets are a valuable tool for processing uncertainty, many decisions and systems require crisp values. For example, when an autonomous car has to slow down, its computer is not able to use a fuzzy concept directly (e.g. slow down *slightly)* but rather needs an actual crisp value. There are several possible ways to reduce a fuzzy set to a crisp value. In fact, any mapping $z : M \to \mathbb{R}$, where M is the space of membership functions, may be considered a defuzzification operator. However, not every mapping would be a useful defuzzification operator. Runkler and Glesner (1993) suggested a set of 13 axioms that a defuzzification operator should satisfy and showed that some of the frequently used defuzzification operators violate some of those axioms. Wierman (1997) also proposed a set of six properties that should be satisfied by defuzzification operators. Roychowdhury and Pedrycz (2001) discussed defuzzification from an epistemological perspective. The following are some commonly used defuzzification operators:

- Centre of gravity (COG):

$$\mathrm{COG}(\mu) = \frac{\int_{\mathbb{R}} x\mu(x)dx}{\int_{\mathbb{R}} \mu(x)dx}; \tag{12}$$

- Median of area (MOA): MOA(μ) is such that

$$\int_{-\infty}^{\mathrm{MOA}(\mu)} \mu(x)dx = \int_{\mathrm{MOA}(\mu)}^{\infty} \mu(x)dx; \quad \text{and} \tag{13}$$

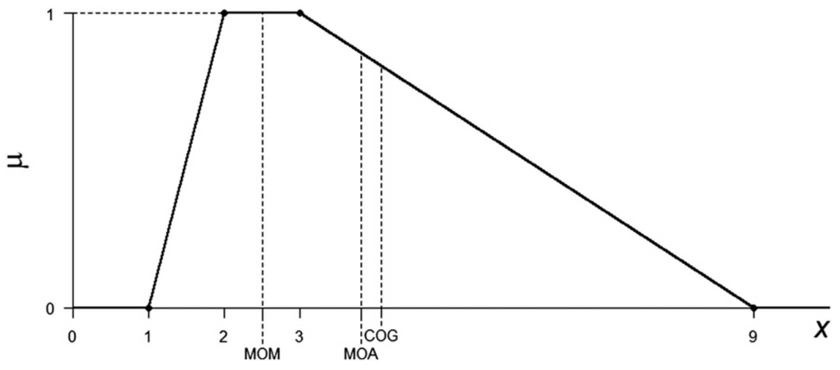

Figure 7: Defuzzification Results of an Asymmetric Trapezoidal Membership Function.

- Mean of maxima (MOM):

$$\mathrm{MOM}(\mu) = \frac{\int_{R(\mu)} x\,dx}{\int_{R(\mu)} dx}, \text{ where } R(\mu) = \left\{x : \mu(x) = \sup_y \mu(y)\right\}. \tag{14}$$

It is assumed that such integrals exist.

For example, let $A = (a, b, c, d, e)$ be a fuzzy set with a trapezoidal membership function. Then,

(1) $\mathrm{COG}(\mu_A) = \frac{(d^2 + cd + c^2 - a^2 - ab - b^2)}{3(d + c - b - a)}$; and

(2) $\mathrm{MOM}(\mu_A) = \frac{b+c}{2}$.

Additionally, if μ_A is symmetric, with $b - a = d - c$, it is possible to show that $\mathrm{COG}(\mu) = \mathrm{MOA}(\mu) = \frac{a+d}{2} = \frac{b+c}{2} = \mathrm{MOM}(\mu_A)$. In other words, for a symmetric trapezoidal membership function, all three operators presented in Eqs. (12) through (14) provide the same result. In general, when μ is a symmetric membership function, trapezoidal or not, it is possible to show that $\mathrm{COG}(\mu) = \mathrm{MOA}(\mu) = z$, where $x = z$ is the axis of symmetry. Figure 7 illustrates the operators for an asymmetric trapezoidal membership function with parameters $A = (1, 2, 3, 9, 1)$. In this case, $\mathrm{COG}(\mu_A) = 4.074$, $\mathrm{MOA}(\mu_A) = 3.804$ and $\mathrm{MOM}(\mu_A) = 2.500$.

Non-probabilistic Entropy: Measuring the Degree of Fuzziness

Much as one quantifies information using the concept of classical entropy, one might be interested in quantifying the degree of fuzziness of a fuzzy set. One of the first attempts to measure the degree of fuzziness was made by Luca and Termini (1972), where they suggested that such a measure, say H, should satisfy the following three requisites:

(1) $H(A) = 0 \Leftrightarrow A$ is a crisp set;
(2) $H(A)$ is maximum if and only if $\mu_A(x) = 0.5, \forall x \in X$; and
(3) $H(A) \geq H(B)$ if $\mu_B(x) \leq \mu_A(x), \forall x \in X : \mu_A(x) \leq 0.5$ and $\mu_B(x) \geq \mu_A(x), \forall x \in X : \mu_A(x) \geq 0.5$.

The first requisite refers to the crispness of a set; since there is no doubt (or fuzziness) about the belonging of an element to a crisp set, the degree of fuzziness must be 0. The second requisite determines the state of highest fuzziness, which occurs when all elements have a membership degree of 0.5 (i.e. the elements belong and do not belong to a set to the same degree). The third requisite states that the more 'sharp' a fuzzy set is (i.e. the closer the membership function is to the membership function of a crisp set), the less doubt one has over the belonging of an element to a set. Consequently, the degree of fuzziness must be smaller. Following these three requisites, Luca and Termini (1972) proposed the following measure of entropy:

$$H(\mu_A) = -K \sum_{i=1}^{n} \left[\mu_A(x_i) \log \mu_A(x_i) + \left(1 - \mu_A(x_i)\right) \log\left(1 - \mu_A(x_i)\right)\right], \tag{15}$$

where K is a positive constant. Later, Ebanks (1983) proposed three additional requisites for entropy; however, Pal and Bezdek (1994) stated that the third additional requisite was not necessary. The remaining two additional requisites are:

(4) $H(\mu_A) = H(1 - \mu_A)$ and
(5) $H(\max\{\mu_A, \mu_B\}) + H(\min\{\mu_A, \mu_B\}) = H(\mu_A) + H(\mu_B)$.

Requisite four concerns the symmetry (around 0.5) of the measure, and it implies that the fuzziness degree of a fuzzy set should be the same as that of its complement. Otherwise, one could just work with the set with a lower fuzziness degree. Finally, using the standard set operations of fuzzy sets, requisite five can be rewritten as $H(\mu_{A \cup B}) = H(\mu_A) + H(\mu_B) - H(\mu_{A \cap B})$; in other words, the degree of fuzziness of a union of fuzzy sets is the sum of their degrees of fuzziness minus the degree of fuzziness of their intersection. The entropy in Eq. (15) satisfies all five requisites. However, several other entropy measures have been proposed. An overview of different entropy measures is presented in Pal and Bezdek (1994).

Fuzzy Numbers

Fuzzy numbers are a specific type of fuzzy sets defined on the real line. They are very useful for representing uncertain concepts in the real line, for example *approximately* 10, a *low* temperature in degrees Celsius, and a *high* price in dollars for a house. A fuzzy number A must satisfy the following properties (Grzegorzewski & Mrowka, 2007; Pedrycz & Gomide, 2007):

(1) $\exists x \in \mathbb{R} : \mu_A(x) = 1$, in other words, the height of A is one;
(2) A is a convex fuzzy set;
(3) A has a bounded support; and
(4) The membership function μ_A is upper semicontinuous.[1]

There are some small differences between various definitions of fuzzy numbers. For example, in Hanss (2005) and Dubois and Prade (1980), property four is replaced by the requirement that μ_A be piecewise continuous. Pedrycz and Gomide (2007) require the core of a fuzzy number to have only one element (i.e. there is only one element, x^*, such that $\mu_A(x^*) = 1$); a fuzzy number with more than one element in its core was named a fuzzy interval.

Despite the abovementioned additional requirements for a fuzzy set to qualify as a fuzzy number, there are still a large variety of possible shapes for the membership functions of fuzzy numbers. A widespread class of fuzzy numbers was introduced in Dubois and Prade (1978, 1980), called L-R fuzzy numbers or flat fuzzy numbers. This class contains several nonlinear shapes in addition to the trapezoidal fuzzy number, which is a special case. The trapezoidal shape is one of the most commonly used shapes for fuzzy numbers. Due to the great importance of trapezoidal fuzzy numbers, several methods have been proposed for using trapezoidal fuzzy numbers to approximate differently shaped fuzzy numbers (Ban & Coroianu, 2014; Grzegorzewski, 2008; Yeh, 2009, 2017).

As is the case with crisp numbers, in some situations one may need to perform algebraic operations on fuzzy numbers. For example, suppose that a company is evaluating a project to build approximately 10 houses and each house will cost around \$200,000 to be built. How much will the company need for the project? Fuzzy arithmetic enables us to operate on fuzzy numbers for the same purposes as performing arithmetic operations on crisp numbers. There are two main approaches for fuzzy arithmetic: the α-cut method, which is based on interval analysis, and the extension principle. In the α-cut method, interval arithmetic is performed at each α-level cut of the fuzzy numbers to obtain the α-cut of the output. The representation theorem (Eq. (10)) provides a way to determine the output fuzzy set from its collection of α-cuts. The α-cut method is easy and fast in computer implementation. However, the α-cut method is based on interval arithmetic, which can lead to the overestimation of uncertainty (Hanss, 2005; Moore & Lodwick, 2003). On the other hand, the extension principle provides a way to generalise functions and operations from the crisp domain to the fuzzy domain. There are many different methods of implementing the extension principle (using different t-norms) and there is no definitive conclusion on the best approach. Fuzzy arithmetic using the extension principle

[1] The requirement of upper semicontinuity is technical. The unfamiliar reader may assume continuity, as most of the fuzzy numbers used in literature have continuous membership functions.

can reduce the problem of overestimation of uncertainty, but it is computationally more expensive.

Membership Function Specification Methods

The membership function is an essential element in fuzzy logic, and a proper specification of this function is fundamental for the successful applicability of any fuzzy-based model. Several different ways to specify membership functions have been proposed. Some of the approaches have expert-based specifications (Pedrycz & Gomide, 1998; Pedrycz & Gomide, 2007), while others are data-driven (Hasuike, Katagiri, & Tsubaki, 2015a, 2015b; Jalota, Thakur, & Mittal, 2017; Kaufmann, Meier, & Stoffel, 2015; Pazhoumand-Dar, Lam, & Masek, 2017; Pedrycz & Wang, 2016; Pota, Esposito, & Pietro, 2013; Runkler, 2016). Although the membership function plays an important role in capturing uncertainty in the construction industry, little research has been done on the specification of membership functions in such contexts. Most of the methods for specification of membership functions in construction applications are expert-based approaches (Ibrahim, Costello, & Wilkinson, 2015; Marsh & Fayek, 2010; Mitra, Jain, & Bhattacharjee, 2010; Prieto, Macías-Bernal, Chávez, & Alejandre, 2017), which are difficult to reproduce and calibrate in different contexts. On the other hand, automatic or data-driven methods, while more objective, have restricted applicability in the construction domain due to their heavy reliance on large datasets, which are often lacking in construction and are not readily transferable from one project context to another (Dissanayake & Fayek, 2008). Selecting a membership function construction technique depends primarily on the type of variable, the method of measurement of that variable, the type and quantity of data available and the availability and knowledge of domain experts. No single technique for generating membership functions is suitable for all applications in construction, and further research is required on how to effectively combine expert-driven and data-driven methods of membership function development and calibration to make them applicable in the construction domain (Dissanayake & Fayek, 2008; Marsh & Fayek, 2010).

Direct Assignment of Membership Functions: Horizontal and Vertical Methods

The horizontal and vertical methods allow an expert or a group of experts to directly provide the values for membership functions. Since this is an expert-driven approach, at least one expert with known competence must be available in order to obtain a suitable membership function. In the horizontal method, the experts are asked to provide a membership value for each $x_i \in X$; these values are then aggregated to obtain a single representative value of membership degree, as follows:

$$\mu_A(x_i) = \sum_{c=1}^{k} w_c \mu_A^{(c)}(x_i), \tag{16}$$

where k is the number of experts, w_c is the weight of the cth expert such that $\sum_{c=1}^{k} w_c = 1$, and $\mu_A^{(c)}(x_i)$ is the membership value of x_i regarding the concept A according to the cth expert. The weight $w = (w_1, \ldots, w_k)$ reflects the relative importance of the experts' opinions. Note that if $\mu_A^{(c)}(x_i)$ is either 0 or 1 and $w_c = k^{-1}$, $c = 1, \ldots, k$, then $\mu_A(x_i)$ is the proportion of experts who believe that x belongs to the concept A. In this case, one could just ask the question to each expert, 'Does x belong to A?' and then calculate the proportion. The weighted arithmetic mean for the aggregation is presented in Eq. (16), but other aggregation operators can also be used.

The vertical method requires that the experts decide which elements in X belong to concept A with a membership of at least $\alpha \in (0, 1]$. In other words, the experts are asked to provide the α-cuts of the fuzzy set under consideration. The different expert opinions must be combined into a unique representative value by using an aggregation operator, such as

$$A_\alpha = \left[\sum_{c=1}^{k} w_c L_\alpha^{(c)}; \sum_{c=1}^{k} w_c U_\alpha^{(c)} \right], \tag{17}$$

where $L_\alpha^{(c)}$ and $U_\alpha^{(c)}$are the lower and upper bounds of the α-cut of the cth expert, respectively. Thus, while the horizontal method requires that experts provide the membership degrees for the elements, the vertical method requires that they provide the elements for each membership degree.

Direct assignment methods are relatively simple to implement, although the methods of collecting experts' opinions must be carefully designed. The vertical and horizontal methods rely heavily on experts' judgements, and due to the subjectivity, context-dependent concepts and different backgrounds of the experts, they can be prone to error. It is fundamental that data collection and the selection of experts are carefully planned to ensure the quality of the data.

Pairwise Comparison Using the Analytic Hierarchy Process

This section describes the process of specifying a membership function by using the analytic hierarchy process (AHP), which was proposed by Saaty (1977, 1980). The AHP method derives ratio scales based on pairwise comparisons and can be viewed as a general theory of measurement (Saaty, 1987). As shown by Pedrycz and Gomide (2007), the AHP method can be used to specify the membership function. In contrast to direct methods, where experts must provide a direct evaluation of the membership function, in this approach, the experts provide relative information by comparing the membership degrees of two elements. Mathematically, for a membership function of a fuzzy set $A \subset X = \{x_1, \ldots, x_n\}$, we build matrix

$$P = \begin{bmatrix} 1 & p_{12} & \cdots & p_{1n} \\ p_{21} & 1 & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & 1 \end{bmatrix}, \tag{18}$$

where $p_{ij} > 0$ represents the intensity with which the membership of x_i is higher than that of x_j with regard to the concept A (i.e. it is an estimate of $\mu_A(x_i)/\mu_A(x_j)$). Saaty (1977) suggested a scale from 1 (equally preferable) to 9 (absolutely more preferable). Other scales can also be used (see Franek and Kresta (2014) and references therein). It is assumed that $\mu_A(x_i) \neq 0$, $\forall i = 1, \ldots, n$. This is not a strong restriction since, if one knows beforehand that the membership of an element is 0, it is not necessary to estimate that membership, and therefore it can be removed from the matrix P. In addition, P is a reciprocal matrix, that is, $p_{ij} = 1/p_{ji}$. Once the matrix P is obtained, the membership values of the elements under consideration are given by the eigenvector associated with the largest eigenvalue of P. An important step when using this approach is to verify the consistency of the data provided by the experts. In the AHP, a matrix P is said to be consistent if $p_{ij}p_{jk} = p_{ik}, \forall i, j, k$. This does not always happen when dealing with data provided by experts. To measure the consistency of a matrix, Saaty (1977) suggested the following inconsistency index:

$$\zeta = \frac{\lambda_{\max} - n}{n - 1}, \tag{19}$$

where $\lambda_{\max}$ is the highest eigenvalue of P. In the same work, it was proved that $\lambda_{\max} = n$ is a necessary and sufficient condition for the consistency of a matrix. Consequently, we want small values of ζ. Pedrycz and Gomide (2007) suggested that we consider a matrix consistent if $\zeta < 0.1$; otherwise, the pairwise comparisons should be reevaluated by the experts to reduce the inconsistency index. Naturally, this is an expert-driven approach.

Statistical Methods

The statistical approaches for determining membership functions are based on probability distributions and are therefore considered data-driven approaches. For instance, in cases where the fuzzy set has a defining feature with a known probability distribution, Civanlar and Trussell (1986) formulated the membership function specification as the following optimisation problem:

$$\begin{aligned} &\text{minimise } 0.5\int_{\mathbb{R}} \mu^2(x)dx \\ &\text{subject to } E_f[\mu] \geq c, \end{aligned} \tag{20}$$

where f is the probability distribution of the defining feature and $c < 1$ is a constant.

Based on the idea of entropy, Hasuike et al. (2015a, 2015b) later proposed that, instead of trying to minimise the size of the fuzzy set like the model described by Eq. (20), one should select the membership function that maximises the entropy presented in Eq. (15) and satisfies the imposed restrictions; additional restrictions can be added to represent information from experts about the membership function (Hasuike et al., 2015b). Many other methods of specifying membership functions based on probability distributions have been studied (Dubois, Foulloy, Mauris, & Prade, 2004; Dubois & Prade, 1986; Geer & Klir, 1992; Pota et al., 2013; Yamada, 2001). In construction applications, Oliveros and Fayek (2005) used the statistical-based method proposed by Dubois and Prade (1986) to determine the membership function of construction activity durations for activity delay analysis.

However, these statistical-based methods require the use of a probability distribution, which may not be viable in practice. In general, one would need a sufficient amount of data to be able to have a reliable estimate of such probability distributions, and these data are frequently not available in construction. Therefore, the applicability of such methods in construction problems is fairly low.

Methods Based on Clustering

An important and frequently used data-driven approach is to determine the membership function based on fuzzy clustering methods, in particular the fuzzy c-means (FCM) algorithm (Bezdek, Ehrlich, & Full, 1984). The FCM algorithm is one of the most popular fuzzy clustering algorithms, and it provides a useful data-driven approach to the specification of membership functions. In contrast to classical clustering approaches, FCM allows data points to belong to different clusters with different degrees of membership. In other words, each cluster represents a concept, and each data point belongs to each concept with a specific membership degree.

Let $x=\{x_1,\dots,x_n\}$ be n data points, where $x_j=(x_{j1},\dots,x_{jd})$ and $j=1,\dots,n$, is a d-dimensional vector, that is, each data point is evaluated according to d features. In FCM, each element $x_j \in X$ can belong to several clusters with different degrees of membership. Assume we have c clusters and let μ be a $c \times n$ matrix where each element $\mu_{ij}, i \le c, j \le n$ represents the degree to which the data point x_j belongs to the ith cluster. Every cluster must have at least one element, that is, $\sum_{j=1}^{n} \mu_{ij} > 0$, and it is required that $\sum_{i=1}^{c} \mu_{ij} = 1$. Both restrictions are related to the concept of fuzzy partition, discussed in the Section Fuzzy Variables and Fuzzy Partitions. Next, the goal is to find a partition that minimises

$$J_m(\mu,\nu)=\sum_{i=1}^{c}\sum_{j=1}^{n}\mu_{ij}^{m}\|x_j-\nu_i\|^2, \tag{21}$$

where ν_i is the centre of the ith cluster and m is a weighting exponent. The parameter m can be viewed as a fuzzification parameter. For $m=1$, the solution μ^* that minimises J_m is necessarily a hard partition, that is, if $\forall \mu_{ij} \in \mu^*$, we have that $\mu_{ij} \in \{0,1\}$. On the other hand, as $m \to \infty$, we have that each coefficient $\mu_{ij} \in \mu^*$

approaches the value c^{-1}, which is the fuzziest state possible. Bezdek et al. (1984) suggested that $m \in [1.5, 3]$ provides good results in most cases. The idea with using FCM to minimise J_m is to iteratively update the pair (μ, ν) from a provided initial partition $\mu^{(0)}$ through the following equations:

$$\nu_i^{(k+1)} = \frac{\sum_{j=1}^{n} \left(\mu_{ij}^{(k)}\right)^m x_j}{\sum_{j=1}^{n} \left(\mu_{ij}^{(k)}\right)^m}, 1 \leq i \leq c \quad \text{and} \tag{22}$$

$$\mu_{ij}^{(k+1)} = \left[\sum_{t=1}^{c} \left(\frac{\|x_j - \nu_i\|}{\|x_j - \nu_t\|}\right)^{\frac{2}{m-1}}\right]^{-1}, \tag{23}$$

where the superscript $(k + 1)$ represents the iteration number. Naturally, many other fuzzy clustering algorithms have been proposed (Chatzis, 2011; Gu, Jiao, Yang, & Liu, 2017; Leski, 2016; Memon & Lee, 2017).

Several different clustering-based techniques for determining membership functions from numerical data are found in the literature (Hong & Lee, 1996; Yadav, H.B., & Yadav, D.K., 2015). As in the case of statistical methods, methods based on clustering require a significant amount of data, often limiting their applicability in the construction industry.

In this section, four commonly used approaches to specifying membership functions were introduced. It is important to note that there is no single technique for generating membership functions that can be generalised across all applications in construction. In fact, a combination of techniques is often more appropriate for developing membership functions for construction modelling. As pointed out by Pedrycz and Vukovich (2002), focusing only on expert-driven approaches may yield fuzzy membership functions with no real evidence to corroborate their usage. On the other hand, focusing exclusively on data-driven approaches may make the interpretation of the membership functions difficult. By combining expert- and data-driven approaches, it is possible to build membership functions with clearer semantics and to calibrate such membership functions to better fit the available data.

Fuzzy Rule-based Systems

Fuzzy rule-based systems, also known as fuzzy inference systems (FISs), provide an approach to representing inexact data and knowledge that is close to human-like thinking. Fuzzy rule-based systems are based on fuzzy rules that employ membership functions to reason or make a decision. They can emulate the human reasoning process within a specific domain of knowledge and make the problem-solving capabilities of an expert available to a non-expert in the field. For example, a traditional expert system may have a rule that states, ‘If rainfall is greater than 2 cm, then productivity is reduced by 25%’. Thus, if rainfall is 1.9 cm, this rule would not activate or fire, and the impact on productivity would not be reflected. However, in reality, a rainfall of 1.9 cm is effectively the same as one of 2 cm in terms of impact on the productivity of a weather-sensitive construction activity. A fuzzy rule would employ linguistic terms in place of crisp numbers and would state: 'If rainfall is *moderate*, then productivity is reduced *somewhat'*. Fuzzy rule-based systems have a clear logic; they are easy to understand; and they are robust in the sense that they react smoothly through the gradual transition of the states of the fuzzy variables in the rules. There are numerous applications of fuzzy rule-based systems in construction (Amiri, Ardeshir, & Zarandi, 2017; Aydin & Kisi, 2015; Debnath, Biswas, Sivan, Sen, & Sahu, 2016; Khamesi, Torabi, Mirzaei-Nasirabad, & Ghadiri, 2015; Kim, H., Kim, K., & Kim, H., 2016; Marsh & Fayek, 2010; Tsehayae & Fayek, 2016).

A fuzzy rule-based system consists of a number of components, as follows:

(1) A knowledge base that consists of a rule base and a database;
(2) An inference engine or decision-making unit;
(3) A fuzzification interface that accepts input from the user; and
(4) A defuzzification interface that provides output to the user.

The knowledge base consists of the database and the rule base. The database is the short-term memory of the system, and it stores the data for each specific task, obtained by interaction with the user or by inference through the fuzzy rule-based system. The rule base is the long-term memory of the system, and it consists of the fuzzy production or expert rules that capture the general knowledge pertaining to the problem's domain. Fuzzy rules are defined in an *if-then* structure, where each variable in the *if* part (also called the input, antecedent or premise) and the *then* part (also called the output, consequent or conclusion) is defined by a set of membership functions representing the linguistic states of the fuzzy variable. For example, suppose one is using two variables, *temperature* and *skill*, each with two levels {*cold*, *warm*} and {*low*, *high*}, respectively, to reason about a third variable, *productivity*, with three levels: *low*, *average* and *high*. Then, a possible rule base is given by

Rule 1:	If	[*Temperature* is *cold*]	and	[*Skill* is *low*]	then	[*Productivity* is *low*]
Rule 2:	If	[*Temperature* is *cold*]	and	[*Skill* is *high*]	then	[*Productivity* is *average*]
Rule 3:	If	[*Temperature* is *warm*]	and	[*Skill* is *low*]	then	[*Productivity* is *average*]
Rule 4:	If	[*Temperature* is *warm*]	and	[*Skill* is *high*]	then	[*Productivity* is *high*]

Note that the variables in the antecedent can be connected by *and* or *or*. In general, the *and* operator (represented by a *t*-norm) can be used when input variables are independent (e.g. temperature and skill), and the *or* operator (represented by an *s*-norm) can be used when input variables are correlated (e.g. size and complexity). When no two rules with the same conditions (antecedent variable states) have different conclusions (consequent variable states), we say that the rule base is consistent. Inconsistent rules result from high (or complete) overlap in the premise and low (or no) overlap in the conclusion.

A serious drawback of the FIS is that the dimension of the model produces an exponential increase in the number of rules necessary to cover all possible combinations of variables. When analysing high-dimensional data, say n fuzzy variables, $X_1, ..., X_n$, each with $p_i \geq 2$ levels, $i = 1, \ldots, d$, the number of rules necessary to cover all possible cases grows exponentially. For example, if we have five input variables, each one with three levels, then it is necessary to have $3^5 = 243$ rules to cover all possible combinations of variables. When a fuzzy ruled-based system covers all possible combinations of variables, it is said to be *complete*.

There are two main approaches to fuzzy inference: Sugeno and Mamdani inference strategies. In Sugeno inference, the consequent functions are crisp functions of the inputs (most frequently constant or linear functions are used). In Mamdani inference, the consequent functions are fuzzy sets, and the overall output of the inference process is a fuzzy set that can be defuzzified to obtain a crisp value. The fuzzification interface accepts input from the user in either numerical or linguistic form and transforms that data into membership values. In the case of crisp input, the crisp values are mapped to the membership function of their respective variables to determine their degree of membership in each rule, which is then used to evaluate the activation strength of the antecedent part of the rules. In the case of linguistic input, the linguistic term that matches the user input receives a membership value of 1, and the other linguistic terms receive membership values at the value they intersect with the user-input linguistic term. Once membership values for all linguistic terms and for all antecedent variables are determined, the antecedent value of each rule is calculated using the appropriate operator (either a t-norm or an s-norm) for each rule.

Rules can be weighted (also referred to as degrees of support (DoS)) ranging from 0 to 1 to indicate their relative importance on the output or decision. If a rule has a weight, the weight is applied to the resulting antecedent value for that rule by multiplying the antecedent value by the rule's weight. The resulting value is then used in rule implication, wherein the value is applied to the consequent membership function representing that rule's output variable(s) state (e.g. *average* productivity) using a t-norm operator, such as *minimum (standard intersection)* or *algebraic product*. Using *minimum*, the rule's output fuzzy set is truncated by the value from the antecedent; using *algebraic product*, the rule's output fuzzy set is scaled by the value from the antecedent. Rule aggregation takes place next, whereby the fuzzy sets that represent the output from each rule are combined by means of an s-norm into a single fuzzy set, yielding one fuzzy set for each output variable that is a combination of all of its output fuzzy sets resulting from rule implication. Common aggregation operators include *maximum (standard union)*, where the maximum value of each output set from each rule is taken for a given variable; *bounded sum*, where the sum of the values of each output set from each rule is taken for a given variable; and *algebraic sum*, where the algebraic sum of the values of each output set from each rule is taken for a given variable. Rule aggregation yields a non-standard membership function shape for each consequent variable. In order to draw conclusions from the fuzzy inference process, the non-standard shape must either be defuzzified (see the Section Defuzzification) to obtain a crisp value or mapped to known standard shapes using a distance measure, such as Euclidean distance. The defuzzification interface provides a crisp value based on the fuzzy output of the inference process.

For example, let $x=(x_1,...,x_n)$ be an input in the following fuzzy rule-based system:

Rule 1:	If	$[X_1$ is $A_{11}]$	and	$[X_2$ is $A_{12}]$	and	...	and	$[X_n$ is $A_{1d}]$	Then	$[Y$ is $B_1]$;
Rule 2:	If	$[X_1$ is $A_{21}]$	and	$[X_2$ is $A_{22}]$	and	...	and	$[X_n$ is $A_{2d}]$	Then	$[Y$ is $B_2]$;
⋮		⋮		⋮				⋮		⋮
Rule k:	If	$[X_1$ is $A_{k1}]$	and	$[X_2$ is $A_{k2}]$	and	...	and	$[X_n$ is $A_{kd}]$	Then	$[Y$ is $B_k]$

Let $w=(w_1,\dots,w_k)$, where $w_i>0$ and $\sum_{i=1}^{k} w_i=1$, be a vector of weights of the rules. Then, since the variables are linked by the word *and*, the activation of rule i is given by $w_i \times T_1\left(\mu_{A_{i1}}(x),\mu_{A_{i2}}(x),\dots,\mu_{A_{id}}(x)\right)$, where $T_1(\cdot)$ stands for a t-norm. If *or* was used, then an s-norm should be used instead of a t-norm. For Mamdani inference, the elements in the consequent $B=\{B_1,\dots,B_k\}$ are themselves fuzzy sets. In this way, the resulting implication of each rule is also a fuzzy set with membership function $\mu_{R_i}(x,y)=T_2\left(w_iT_1\left(\mu_{A_{i1}}(x),\mu_{A_{i2}}(x),\dots,\mu_{A_{id}}(x)\right),\mu_{B_i}(y)\right)$, where T_2 is a t-norm. Next, the results from each rule must be aggregated, resulting in a fuzzy set with a membership function given by $\mu_R(x,y)=S\left(\mu_{R_1},\dots,\mu_{R_k}\right)$, where S represents an s-norm. Lastly, one may need to defuzzify the output fuzzy set to draw conclusions. In the Sugeno inference system, the elements $B=\{B_1,\dots,B_k\}$ are functions $f(x_1,\dots,x_d,y)$. Then, for Sugeno, the result of each rule is given by $R_i=w_iT_1\left(\mu_{A_{i1}}(x),\mu_{A_{i2}}(x),\dots,\right.$ $\left.\mu_{A_{id}}(x)\right)f(x_1,\dots,x_d,y)$ and the final output is given by $R=\sum_{i=1}^{k}\frac{R_i}{\dot{w}}$, where $\dot{w}=\sum_{i=1}^{k}$ $w_iT_1\left(\mu_{A_{i1}}(x),\mu_{A_{i2}}(x),\dots,\mu_{A_{id}}(x)\right)$.

An important aspect of fuzzy rule-based systems is that any point $x\in X$ should be covered by a fuzzy set in at least one rule; in other words, the activation of at least one rule must be greater than zero for any given point in the universe of discourse. A universe of discourse that is not well covered may result in the fuzzy rule-based system failing to cover relevant regions of the input space. On the other hand, having a comprehensive model with a complete rule base does not necessarily mean that all of the information it provides is useful or significant to the output, and its complexity may make it difficult for a decision maker to identify the most significant factors affecting the output. Therefore, there is often a need to reduce the number of input variables by creating rule blocks of variables that belong together (e.g. weather-related variables). The output of each rule block then becomes the input in the next rule block in the hierarchical level. There are a number of approaches for addressing this issue. When data are not available, the variables can be categorised based on their similarity (e.g. all weather-related factors impacting productivity), and sub-models can be created for each group of factors, wherein the output of a sub-model is an intermediate factor (e.g. weather) that is then used with other intermediate factors (e.g. crew characteristics) to predict the output of the fuzzy rule-based system (e.g. productivity). Thus, the number of rules in a given rule block can be maintained at a manageable size.

If data are available for input and output variables in the fuzzy rule-based system, a number of techniques are available for developing the rule blocks. One approach is to use a neural network that maps inputs to outputs in order to derive the rules' DoS and establish a threshold DoS value below which rules are eliminated. The other approach is to preprocess the data using statistical methods, such as correlation analysis. In this approach, input variables that are highly correlated to each other can be evaluated in terms of how highly they are correlated to the output. Variables that are not highly correlated to the output but highly correlated to other input variables that are highly correlated to the output can be eliminated. Additionally, the strength and direction of correlation is useful in establishing the linguistic terms of the rules. For example, a strong positive correlation indicates that when the input is large, the output is also large. A strong negative correlation would indicate that when the input is large, the output is small. A moderate positive correlation would indicate that when the input is large, the output is medium. Given the choice of operators in a fuzzy rule-based system, a sensitivity analysis is often performed with model validation to choose the best combination of fuzzy operators.

When data are available, a subset of data can be used for model development, and the rest retained for model validation. By using the data for model development, one can search for the best possible configuration for the data in hand. In this way, the number of levels in a fuzzy variable can be increased or decreased, and the shapes of the membership functions, the number of rules in the system, the aggregation operator, the defuzzification method, and the *t*-norms or *s*-norms used can all be adjusted to obtain the best possible model. However, fitting the data in hand alone might not be enough. Frequently, the objective is to construct a model that is able to provide an accurate answer for situations other than the one presented by the data used in the construction of the model. For example, if by providing a different input, such as a different combination of temperature, crew size and project size that was not present in the data used to build the model, a model has poor performance, then its usefulness for guiding decisions in a new context in construction is limited.

In order to develop models, the data are usually divided into three sets, one for training (fine tuning the parameters of) the model, one for validation (checking how the model performs with unseen scenarios/data) and one test set for estimating the accuracy of the model. The validation set is not used to estimate the accuracy of the model because the validation set is used to choose the best possible configuration of the model; using this same set would overestimate the accuracy of the model. Therefore, a separate dataset that is not used at all for modelling purposes is used to estimate the accuracy of the model. Accuracy can be estimated according to several different measures depending on the problem. For example, let $x' = (x_1, \ldots, x_n)$

be n data points and $y' = (y_1, \ldots, y_n)$ be the corresponding values of the output variable. When trying to predict a continuous variable, one can use the mean squared error (MSE):

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2, \tag{24}$$

where $\hat{y}_i$ is the output of the fuzzy rule-based system for the input x_i. In addition to validating and testing of the model, it is important to perform a sensitivity analysis. The sensitivity analysis evaluates how the model behaves with some small variations in input values or in parameters (e.g. the parameters of the fuzzy sets in the rules, the t-norm [or s-norm] used in the activation, the weights of the rules, or the defuzzification operator). It is expected that small changes in the input/parameters result in small changes in the output of the model. This is important because in practice the input is usually subject to small fluctuations due to uncertainties such as subjectivity or randomness. Models that are very unstable in this sense are unreliable in many situations, unless it is possible to guarantee that the inputs are completely error-free, which is extremely uncommon in practice.

Methods of Generating Fuzzy Rule-based Systems

In a fuzzy rule-based system, it is not an easy task to determine the rules, the number of levels of each fuzzy variable and the membership function of each of these levels. Therefore, methods to support the specification of a fuzzy rule-based system are necessary. Many methods have been proposed (Cintra, Camargo, & Monard, 2016; Gou, Hou, Chen, Wang, & Luo, 2015; Khamesi et al., 2015; Tang et al., 2012). A common approach is to use FCM. In the Section Membership Function Specification Methods, FCM was used to determine the membership function of a fuzzy set. However, it is also possible to use FCM to determine membership functions and fuzzy rules at the same time. The idea is to consider the membership of each of the c clusters as the membership of each of the rules. For example, let $x' = (x_1, \ldots, x_n)$ be n data points and $y' = (y_1, \ldots, y_n)$ be the corresponding values of the output variable. One approach is to combine the data into $z = (x, y)$ and apply FCM, in this way obtaining the membership function of each one of the c clusters. Then, for a new data point x^*, the membership function of the consequent of the ith rule is given by $\mu_i(y^*) = \mu_{C_i}(x^*, y^*)$, where μ_{C_i} is the membership function of the ith cluster provided by FCM.

Fuzzy Hybrid Modelling in Construction

Fuzzy logic has proven to be a valuable tool for modelling and processing certain types of uncertainties (e.g. subjectivity, ambiguity and vagueness) that are not easily modelled by statistical methods. The capabilities of fuzzy logic, especially the ability to model linguistic terms and to reason with non-probabilistic uncertain concepts, provide an effective way of handling information provided by experts, which is a major advantage for the construction industry due to its heavy reliance on experts' judgements. However, as with any other methodology, fuzzy logic alone has limitations and is not able to address every single issue in construction. At the same time, most of the classical modelling approaches in construction are often limited in their ability to model and reason in the presence of subjectivity and vagueness. Therefore, developing innovative approaches that combine classical methodologies with fuzzy logic to obtain hybrid methods that are able to effectively handle the multiple facets of construction problems is of great importance.

For example, fuzzy logic can be combined with machine learning techniques such as logistic regression, artificial neural networks and evolutionary- and swarm-based algorithms (Bouhoune, Yazid, Boucherit, & Chériti, 2017; Mirahadi & Zayed, 2016). Machine learning is particularly valuable in contexts with numerous interacting factors that are difficult to assess but that have a significant impact on project outcomes and construction decision-making. However, current applications of machine learning techniques in construction are limited, as these techniques require adequate numerical data and have a limited ability to capture subjectivity, ambiguity and vagueness. Fuzzy logic, on the other hand, can handle subjectivity, ambiguity and vagueness, but it is unable to learn from data. Integrating fuzzy logic with machine learning techniques can produce new approaches with the functionality necessary to overcome the limitations of each individual technique. Fuzzy machine learning techniques enable the integration of knowledge- and data-driven approaches and yield models that better suit the nature of construction decision-making (Hüllermeier, 2015).

Another example of fuzzy hybrid techniques used for construction process modelling is the integration of fuzzy logic with simulation methods such as discrete event simulation, system dynamics and agent-based modelling. Existing simulation methods rely on the availability of adequate numerical data and do not account for the non-probabilistic uncertainties that exist in real-life situations, both in variables and in their relationships. By integrating fuzzy logic with simulation, the capabilities of these models can be enhanced to allow them to effectively address real-life problems that exhibit both random and subjective uncertainty (de Salles, Neto, & Marujo, 2016; Gerami Seresht & Fayek, 2015; Nojedehi & Nasirzadeh, 2017; Raoufi & Fayek, 2015; Sadeghi, Fayek, & Mosayebi, 2013; Sahebjamnia, Tavakkoli-Moghaddam, & Ghorbani, 2016; Song, J., Song, D., & Zhang, 2015). Moreover, integration allows such models to handle project contexts where there may be inadequate numerical data.

Since the use of fuzzy logic in the construction domain as a stand-alone technique presents certain obstacles, much research has been done to better model construction problems using fuzzy hybrid techniques. The chapter 'Overview of Fuzzy Hybrid Techniques in Construction Engineering and Management' presents an overview of applications of fuzzy hybrid techniques in construction engineering and management, and it describes future research directions for dealing with some of the limitations of existing fuzzy hybrid techniques in this field.

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